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# On the validity of certain classical assumptions in aircraft stability

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# ON THE VALIDITY OF CERTAIN CLASSICAL ASSUMPTIONS IN AIRCRAFT STABILITY

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# ON THE VALIDITY OF CERTAIN CLASSICAL ASSUMPTIONS IN AIRCRAFT STABILITY

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## SUMMARY

The purpose of this report was the investigation of the symmetric modes of motion of an airplane and of the concepts which have been used to obtain relatively simpler expressions for the period of these motions. The utility of the Reeves Electronic Analog Computer in this study was investigated.

Specifically, the concepts concerning the symmetric motions were applied to the general body axes equations of motion in different ways. After the application of these simplifying assumptions, the modified equations were solved for the period of the motion concerned. Numerical data for a representative jet fighter type aircraft at 40,000 feet, Mach number .75, and at sea level, Mach number .25 was utilized in the solution by the different methods. These values for the period were compared to the true values as determined from the complete equations of motion.

The possibility of increasing the accuracy of the period obtained in one of the methods was examined.

A major objective of this report was the investigation of the utility of the analog computer in solving problems of this type. Mathematical treatment, while not difficult, is cumbersome and slow. The analog computer, on the other hand, after having been supplied with information, achieves a solution almost instantaneously. It was desired to examine the effects of changed stability derivatives and coupling between the equations utilizing the computer.

The conclusions reached as a result of this investigation are:

1. The concepts concerning the short period symmetric motion of an airplane when applied to the general equations provide an acceptable approximation to the period of that motion.
2. Concepts concerning the period of the phugoid mode, when applied, provide accuracies ranging from 30 percent low to 12 percent high, depending upon the method of application. It was concluded, therefore, that use of the existing simplifications does not accurately describe the period of the true motion because essential couplings are neglected.
3. The Reeves Electronic Analog Computer provided rapid accurate solutions to many sets of differential equations which had to be solved. The utility of the computer in investigations such as this in unquestionable. Variation of stability derivatives or coupling between equations could be accomplished by changing one or several resistances and the new solution was immediately obtainable.

This study was conducted at the University of Michigan, Ann Arbor, Michigan, during the period February to June 1955 under the supervision of Professors M. A. Brull and J. D. Schetzer of the Aeronautical Engineering Department.

# INTRODUCTION

The advent of flexible high speed aircraft has somewhat complicated the problems which must be solved by the aerodynamicist and the structures specialist. With these complications has come an increasing tendency to write the equations of motion of the airplane in the body axes system of coordinates.

The purpose of this report was the investigation of the symmetric modes of motion of a representative modern airplane in the body axes system and in particular, the investigation of the validity of the simplifying assumptions which have been used to obtain rapid approximate solutions for the period of these motions.

The assumptions which have been made are based upon the observations of Lanchester who was the first to call the long period motion the phugoid. Lanchester stated that phugoid motion takes place at essentially constant angle of attack and further that the pitching motion is so slow that the aerodynamic moment may be considered zero, or nearly so. When these assumptions are applied to the wind axes equations of motion and additional terms which do not materially affect the period are ignored, an expression is obtained for the period of the motion of .138 V. The same results have been obtained from considerations of kinetic and potential energy interchange.

The current literature does not contain examples of the application of the phugoid simplifying assumptions in the body axes equations of motion.

The short period mode of motion is defined as a motion in pitch of very short duration which takes place as the angle of attack, which has been changed due to a disturbance, returns to the equilibrium value. The motion is so rapid that it is assumed that the forward speed does not change and consequently, the motion is independent of changes in perturbations in the forward speed.

## SYMBOLS

$V_p$ aircraft velocity ft/sec

$X$ aerodynamic force in X body axis direction lbs

$Z$ aerodynamic force in Z body axis direction lbs

$M$ aerodynamic moment about the Y body axis ft lbs

$u$ perturbation velocity in X body axis direction ft/sec

$w$ perturbation velocity in Z body axis direction ft/sec

$q$ perturbation velocity in Y body axis rad/sec

$\cdot$ dot over a symbol - derivative of symbol with respect to time

$\lambda$ operator for derivative with respect to time, $\frac{d}{dt}$

$X_u$ dimensional derivative of the aerodynamic force in the X body axis direction with respect to the perturbation velocity u; $\frac{\partial X}{\partial u}$ lb sec/ft

$X_w$ $\frac{\partial X}{\partial w}$ lb sec/ft

$Z_u$ $\frac{\partial Z}{\partial w}$ lb sec/ft

$Z_w$ $\frac{\partial Z}{\partial w}$ lb sec/ft

$Z_q$ $\frac{\partial Z}{\partial q}$ lb sec/ft

$M_u$ $\frac{\partial M}{\partial u}$ ft lb sec/ft

$M_w$ $\frac{\partial M}{\partial w}$ ft lb sec/ft

$M_{\dot{w}}$ $\frac{\partial M}{\partial \dot{w}}$ ft lb sec/ft

$M_q$ $\frac{\partial M}{\partial q}$ ft lb sec/radian

$\alpha$ angle of attack of M.A.C. relative to wind - degrees

$\Theta_1$ angle of pitch of X axis measured from the horizon - degrees

$U_1$ steady state linear velocity along X axis - ft/sec

$W_1$ steady state linear velocity along Z axis - ft/sec

m mass in slugs

$I_{yy}$ moment of inertia about Y body axis - slug ft

M Mach number

$C_L$ Lift coefficient

$F_X$ Sum of forces in X body axis direction

# SYMMETRIC EQUATIONS OF MOTION

## (BODY - AXES SYSTEM)

The six equations of motion break down into two sets of three equations. One set describes asymmetric motion. The other set describes symmetric motion. The linearized equations for symmetric motion, neglecting $X_q$, in body axes coordinates are:

$$F_x = 0 = [X_u - m\lambda]u + [X_w]w + [-mW_1 - \frac{mg\cos\Theta_1}{\lambda}]q$$

$$F_z = 0 = [Z_u]u + [Z_w - m\lambda]w + [Z_q + mU_1 - \frac{mg\sin\Theta_1}{\lambda}]q$$

$$M_y = 0 = [M_u]u + [M_w - M_{\dot{w}}\lambda]w + [M_q - I_{yy}\lambda]q$$

The determinant of the coefficients is set equal to zero and the stability quartic is obtained

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

where

$$A = 1$$

$$B = -\frac{Z_w}{m} - \frac{X_u}{m} - \frac{M_q}{I_{yy}} - \frac{M_{\dot{w}}}{mI_{yy}}(Z_q + mU_1)$$

$$C = \frac{M_q}{mI_{yy}}(Z_w + X_u) + \frac{W_1}{mI_{yy}}(M_{\dot{w}}Z_u + M_u) + \frac{M_{\dot{w}}}{I_{yy}} g\sin\Theta_1$$

$$+ \frac{Z_q + mU_1}{m^2 I_{yy}}(M_{\dot{w}}X_u - M_w) + \frac{1}{m^2}(X_u Z_w - Z_u X_w)$$

$$D = \frac{M_q}{m^2 I_{yy}}(Z_u X_w - X_u Z_w) - \frac{g\sin\Theta_1}{mI_{yy}}(M_w Z_u - M_u Z_w) + \frac{W_1}{mI_{yy}}(M_w Z_u - M_u Z_w)$$

$$- \frac{(Z_q + mU_1)}{m^2 I_{yy}} \frac{X_w M_u}{m^2 I_{yy}} - X_u M_w + \frac{g\cos\Theta_1}{mI_{yy}}(M_{\dot{w}}Z_u + mM_u)$$

$$E = \frac{g\cos\Theta_1}{mI_{yy}}(M_w Z_u - M_u Z_w) + \frac{g\sin\Theta_1}{mI_{yy}}(X_w M_u - M_w X_u)$$

The equations of motion are developed in Ref. 1.

When the analog computer is to be utilized, it is convenient to rewrite the equations of motion as follows:

$$m\dot{u} = X_u u + X_w w - m\,W_1 q - Mg \cos\Theta_1 \int q$$

$$m\dot{w} = Z_u u + Z_w w + [Z_q + mU_1]_q - mg \sin\Theta_1 \int q$$

$$I_{yy}\dot{q} = M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q$$

The computer circuit for the solution of these equations is included as Fig. 1. The airplane numerical data supplied to the circuit is controlled by positioning the potentiometers. Each potentiometer controls a specific airplane parameter. These are:

| | | |
|---|---|---|
| 1. $m$ | 6. $X_w$ | 11. $M_u$ |
| 2. $X_u$ | 7. $mW_1$ | 12. $Z_u$ |
| 3. $m$ | 8. $mg \sin\Theta_1$ | 13. $Z_q + mU_1$ |
| 4. $Z_w$ | 9. $M_w$ | 14. $M_q$ |
| 5. $I_{yy}$ | 10. $M_{\dot{w}}$ | 15. $mg \cos\Theta_1$ |

## APPLICATION OF CLASSICAL ASSUMPTIONS FOR SIMPLIFICATION OF THE SOLUTIONS TO THE MODES OF MOTION

The roots of the stability quartic yield the characteristics of the short period and phugoid motions. Various simplifying assumptions concerning these motions have been introduced in the past. Accuracy obtainable after the assumptions have been applied varies. Normally, the short period motion is so heavily damped and so rapid that it is of little consequence. It does become important if the period is within the normal human response lag and not heavily damped, primarily because a pilot may inadvertently reinforce the motion when he attempts to correct it. The short period mode is important when platform stability is a consideration. The phugoid motion is a long period slow oscillation with weak damping and of such little consequence that it may be negatively damped and not render an airplane unacceptable. Because of the apparent unimportance of these modes in piloted subsonic aircraft, approximations to their period and damping are justified and desirable.

However, the importance of these modes of motion in pilotless missiles at very high speeds has not been completely investigated. When a relatively long slender missile encounters severe turbulence or a sharp gust, the possibilities seem to be present for considerable control difficulties. These motions could intensify the difficulties.

It follows then that it is important to have a method for the rapid, accurate determination of the symmetric modes.

## SHORT PERIOD METHOD

The short period motion is obtained by assuming that the motion proceeds at constant forward velocity ($\dot{u} = 0$). This follows if it is reasoned that by the time the forward velocity has changed, the motion has damped out and has ceased to exist. The assumption permits a reduction of the equations of motion to essentially a coupling between the perturbations w and q. The equations of motion become:

$$F_z = 0 = [Z_w - m\lambda]w \quad + \quad [Z_q + mU_1 - \frac{mg \sin \Theta_1}{\lambda}]q$$

$$M_y = 0 = [M_w + M_{\dot{w}}\lambda]w \quad + \quad [M_q - I_{yy}\lambda]q$$

Solution of these equations as written involves the solution of a cubic. The equation becomes a quadratic if the perturbation gravity force is assumed very small and ignored. The quadratic is:

$$A\lambda^2 + B\lambda + C = 0$$

where:

$$A = 1$$

$$B = -\left[\frac{M_q}{I_{yy}} + \frac{Z_w}{m} + \frac{M_{\dot{w}}}{mI_{yy}} \quad (Z_q + mU_1)\right]$$

$$C = \left[\frac{Z_w M_q}{mI_{yy}} \quad \frac{M_w}{mI_{yy}} (Z_q + mU_1)\right]$$

The utility of the analog computer is demonstrated by the fact that the computer circuit for the short period method is achieved by uncoupling potentiometers 11 and 12 in Fig. 1. This uncouples and removes from the circuit all contribution to the short period of the u perturbation velocity and the solution is obtained using the reduced Z force and moment equations.

Ref. 2 shows a simplified short period method in the wind axes system.

## PHUGOID METHOD

The long period phugoid motion is usually obtained by applying Lanchester's original assumptions. Ref. 3 states that in a phugoid motion the angular pitching movement of the aircraft is at all times very slow and this implies that the aerodynamic pitching moment is nearly zero. Hence, the angle of attack is at all times very nearly that corresponding to zero pitching moment. This implies nearly constant angle of attack. Ref. 3 further assumes that $\alpha$ and $C_L$ is strictly constant.

This investigation revealed that a choice is available for the method of enforcing zero change in angle of attack in the body axes equations.

By definition:

$$\alpha = \arctan \frac{W_1}{U_1}$$

$$d\alpha = \frac{\partial \alpha}{\partial W_1} dW_1 + \frac{\partial \alpha}{\partial U_1} dU_1$$

if $dW_1$ = perturbation velocity in Z direction = w

and $dU_1$ = perturbation velocity in X direction = u

then

$$\Delta\alpha = \frac{\partial \alpha}{\partial W_1} w + \frac{\partial \alpha}{\partial U_1} u$$

$$\Delta\alpha = \frac{w \cos \alpha - u \sin \alpha}{V_p}$$

## PHUGOID METHOD I

The expression for the perturbation angle of attack is usually simplified by assuming the equilibrium angle of attack is small, and

because of this assumption, that

$$\Delta\alpha \cong \frac{w}{V_p}$$

By this method, the perturbation $\alpha$ is related directly to the perturbation velocity w.

When Lanchester's assumptions are applied to the equations of motion based on this near equality for $\Delta\alpha$, and $\Delta\alpha = w = 0$, the stability quartic reduces to a quadratic which gives an approximate solution. The quadratic is obtained by ignoring the moment equation and deleting all terms in w in the force equations. Two equations remain in the variables u and q. These are:

$$F_x = 0 = [X_u - m\lambda]u \quad - \quad [mW_1 + \frac{mg\cos\Theta_1}{\lambda}]q$$

$$F_z = 0 = \quad [Z_u]\,u \quad + [Z_q + mU_1 - \frac{mg\sin\Theta_1}{\lambda}]q$$

The determinant of the coefficients set equal to zero yields:

$$A\lambda^2 + B\lambda + C = 0$$

Where:

$$A = [Z_q + mU_1]$$

$$B = -[X_u\left(\frac{Z_q}{m} + U_1\right) + mg\sin\Theta_1 + Z_uW_1]$$

$$C = g\,[X_u\sin\Theta_1 - Z_u\cos\Theta_1]$$

When the analog computer is utilized, the two equations are rewritten as follows:

$$m\ddot{u} = X_u\dot{u} - mW_1\dot{q} - mg\cos\Theta_1 q$$

$$(Z_q + mU_1)\dot{q} = mg\sin\Theta_1 q - Z_u\,\dot{u}$$

The computer circuit is shown in Fig. 2. Specific airplane parameters are controlled by the potentiometers as follows:

1. $Z_q + mU_1$      2. $mg\sin\Theta_1$

5. m 6. $Z_u$

7. $X_u$ 8. mg cos $\Theta_1$

9. $mW_1$

Switch in Fig. 2 is open.

This method of simplification produces a period which is greater than the true period.

PHUGOID METHOD II

A second method for the reduction for the complete equations to obtain the period of the phugoid mode was developed by the writers during this investigation. This method involves the application of the identical simplifying assumptions of method I, but the complete equality

$$\Delta\alpha = \frac{w \cos \alpha - u \sin \alpha}{V_p} = 0$$

was used to enforce no change in angle of attack. Then $w = u \tan \alpha$ was substituted in the equations and the third equation was ignored. The three equations reduce to two equations which are:

$$F_x = 0 = [X_u - m\lambda + X_w \tan \alpha]u - [m W_1 + \frac{mg \cos \Theta_1}{\lambda}]q$$

$$F_z = 0 = [Z_u + Z_w \tan\alpha - m\lambda \tan\alpha]u + [Z_q + mU_1 - \frac{mg \sin\Theta_1}{\lambda}]q$$

The determinant of the coefficients set equal to zero yields:

$$A\lambda^2 + B\lambda + C = 0$$

where:

$$A = m\ [U_1 + W_1 \tan \alpha + \frac{Z_q}{m}]$$

$$B = -\ [(Z_q + mU_1)(\frac{X_u}{m} + \frac{X_w}{m} \tan \alpha) + mg\ (\sin \Theta_1 - \cos \Theta_1 \tan \alpha)$$

$$+ W_1\ (Z_u + Z_w \tan \alpha)]$$

$$C = g\,[\sin\Theta_1\,(X_w \tan\alpha + X_u) - \cos\Theta_1\,(Z_u + Z_w \tan\alpha)]$$

The equations are rewritten as follows when the analog computer is used:

$$m\ddot{u} = (X_u + X_w \tan\alpha)\,\dot{u} - m\,W_1\dot{q} - mg\cos\Theta_1\,q$$

$$(Z_q + m\,U_1)\dot{q} = mg\sin\Theta_1\,q - (Z_u + Z_w\tan\alpha)\,\dot{u} + m\ddot{u}\tan\alpha$$

The computer circuit is shown in Fig. 2. Specific airplane parameters are controlled by the potentiometers as follows:

1. $Z_q + mU_1$
2. $mg \sin \Theta_1$
3. $m$
4. $m \tan \alpha$
5. $m$
6. $Z_u + Z_w \tan \alpha$
7. $X_u + X_w \tan \alpha$
8. $mg \cos \Theta_1$
9. $mW_1$

Switch in Fig. 2 is closed.

This method of simplification produces a period which is less than the true period.

Solution by Method II provides a period which is exactly equal to .138 $V_p$, arrived at in wind axes in Ref. 2, and is exactly equal to $4.44 \frac{V_p}{g}$ derived in Ref. 3. Thus Method II is the true application of Lanchester's Basic Assumptions in body axes. These assumptions are based on the pure interchange of kinetic and potential energy.

Ref. 4 states, "To secure a lift proportional to the square of the speed, the wings must exert a constant lift coefficient, so that their incidence must be constant. This implies that the aeroplane is statically stable as regards pitching, that it has a negligible moment of inertia about the pitching axis, and the length between wings and tail is small compared to the radii of curvature of the flight path."

Method II does not substantiate this statement since substitution of $w = u \tan \alpha$, (for the condition $\Delta\alpha = 0$) in the moment equation causes $M_w$ and $M_u$ to be grouped in the same coefficient and these terms exactly cancel each other because they become equal in magnitude and opposite in sign. ($|M_u| = |M_w \tan \alpha|$). This leaves the moment equation without a static stability contribution and produces the same effect as zero static stability. In fact, solutions of pairs of the simplified equations with the moment equation included as one of the pair show the motion to be aperiodic. This is the motion to be expected from an airplane with zero static stability. The condition of zero static stability may be solved quickly on the analog computer by positioning the two potentiometers which control $M_u$ and $M_w$ on the total airplane circuit to zero. Computer solutions for this case show aperiodic motion.

It is apparent that the Lanchester idealization enforces neglect of the moment equation because as far as the phugoid mode is concerned, the moment equation becomes inconsistent with the two force equations. This requires that the two force equations be used to obtain the solution for the phugoid mode and that an error in period must be accepted.

## MODIFIED PHUGOID METHOD II

It should be noted that $\Delta\alpha \cong \frac{w}{V_p} = 0$, if applied in the body axes equations, produces a period which is larger than the true period; whereas, Method II produces a period which is smaller than the true period.

Comparison of the determinantal coefficients resulting from both $\Delta\alpha$ approximations was made and the terms in each of the coefficients which largely effected the period were examined. It was noted that the gravity

contribution in the coefficient multiplying $\lambda$ to the first power (Method II) was completely destroyed by tan $\alpha$.

$$[mg\ (\sin \Theta_1 - \cos \Theta_1 \tan \alpha)] \qquad \alpha \cong \Theta_1$$

The classical explanation of phugoid motion is based on the interchange of kinetic and potential energy. The complete removal of all effects of gravity, therefore, seems illogical. In short, if the factor tan $\alpha$ is removed from the term $m\lambda \tan \alpha$ (second equation), the gravity effects are restored and the solution for the period of the phugoid is accurate to within one percent. The resultant period is nearly the actual value.

This result is an experimental result arrived at on physical grounds alone and perhaps may not be applied with such accuracy in general because it cannot be mathematically justified. As shown above, the idealizations of Lanchester as generally applied are also not mathematically justified. (When $\Delta\alpha = 0$, the airplane is not simultaneously statically stable in the equations of motion.) If the additional reasoning concerning the gravity term is accepted as described on physical grounds, the coefficients of the quadratic describing the phugoid mode are:

$$A = m\left[U_1 + W_1 + \frac{Z_q}{m}\right]$$

$$B = -\left[(Z_q + mU_1)\left(\frac{X_u}{m} + \frac{X_w}{m}\tan\alpha\right) + mg\ (\sin \Theta_1 - \cos \Theta_1) + W_1\ (Z_u + Z_w \tan \alpha)\right]$$

$$C = g\left[\sin \Theta_1\ (X_u + X_w \tan \alpha) - \cos \Theta_1\ (Z_u + Z_w \tan \alpha)\right]$$

The computer solution for Modified Method II is obtained by changing the setting of potentiometer 4 in Fig. 2.(switch closed) to remove the factor tan $\alpha$.

Numerical examples for Method I, Method II, and Modified Method II are included in this report. Method I is 12 percent high, Method II is seven percent low and Modified Method II is within one percent, all referred to the actual period.

## PHUGOID METHOD III

One other method for an approximate solution for the phugoid mode has been used. This method makes use of the fact that phugoid motion is essentially a coupling between u and $\int q$. The method of simplification used is to delete the w coupling terms in the force and moment equations thus uncoupling the Z force equation from the system. The X force equation and the moment equation thus reduced are solved simultaneously. The equations are:

$$F_x = 0 = [X_u - m\lambda]u - [mW_1 + \frac{mg \cos \Theta_1}{\lambda}]q$$

$$M_y = 0 = [\ M_u\ ]u + [\ M_q - I_{yy}\lambda\ ]q$$

The resulting cubic equation is:

$$A\lambda^3 + D\lambda^2 + C\lambda + D = 0$$

where:

$$A = 1$$

$$B = -\left[\frac{m\ M_q + I_{yy}\ X_u}{m\ I_{yy}}\right]$$

$$C = \left[\frac{X_u\ M_q + m\ W_1\ M_u}{m\ I_{yy}}\right]$$

$$D = \left[\frac{M_u\ g\ \cos \Theta_1}{I_{yy}}\right]$$

This method produces an error in the period of 25 to 30 percent.

Solutions on the Reeves Electronic Analog Computer show this error. The computer solution is obtained by deleting the connections to potentiometers 6, 9, and 10 in Fig. 1. This effectively uncouples the Z force equation. The most powerful term in reducing the period was determined to be $[M_w]w$. This was shown by step by step replacement of the deleted terms. When $[M_w]w$ alone was reconnected, the period was restored to the correct value.

One other idealization was applied to the equations of motion in an attempt to find a simple accurate representation of the phugoid mode. Because the phugoid is described as a long slow oscillation about a datum line, it seemed reasonable to assume that motions of this type would be characterized by very small accelerations in the direction normal to the flight path. This may be expressed as:

$$\dot{w} - U_1 q = 0$$

This equation was solved for q and substitution for q was made in the equations of motion. Pairs of the equation were solved with varying results. Solution of the two force equations produced aperiodic motion; solution of the Z force equation and the moment equation produced a short period mode frequency; and solution of the X force equation and the moment equation produced a cubic. This cubic was not solved but known accurate values of the phugoid roots were substituted to ascertain whether or not the correct phugoid mode was contained in the cubic. It was found that the roots did not satisfy the equation even remotely and the method was discarded.

It seemed logical to assume further that if the normal accelerations are small or zero, that the condition of zero angle of attack change would occur simultaneously. When this additional condition was imposed, the equations of motion reduced to three equations in u but all the solutions indicated aperiodic motion.

## NUMERICAL RESULTS AND DISCUSSION

The following examples are selected to facilitate comparison of the results obtainable by the several methods discussed above. The airplane upon which this is based is a representative jet fighter type aircraft. Two case are presented: first, the case of Mach number 0.75 at 40,00 feet;

second, the case of Mach number 0.25 at sea level. Normal mathematical solutions are presented and then are verified by Reeves Analog Computer solutions for the first case. The second case will be presented as analog computer solutions only.

CASE I

AIRPLANE DATA AND VELOCITIES

| | |
|---|---|
| $S = 600$ sq ft | $X_u = -5.19$ lb sec/ft |
| $m = 1180$ slugs | $X_w = 48.8$ lb sec/ft |
| $I_{yy} = 80{,}000$ slug ft$^2$ | $Z_u = -70.8$ lb sec/ft |
| $h = 40{,}000$ ft altitude | $Z_w = -807$ lb sec/ft |
| $\alpha = 2.48$ degrees $= \Theta_1$ | $M_w = -637$ lb sec |
| $V_p = 728$ ft/sec | $M_{\dot{w}} = 10.08$ lb sec$^2$ |
| $U_1 \cong 728$ ft/sec | $M_u = 27.5$ lb sec |
| $W_1 = 31.5$ ft/sec | $Z_q = -4.94 \times 10^3$ lb sec |
| $M = 0.75$ | $M_q = -128.4 \times 10^3$ lb ft sec |

Using this data, the stability quartic coefficients are:

$A = 1$
$B = 2.2054$
$C = 6.8699$
$D = .04604$
$E = .02291$

By Bairstow's approximate factorization and one iteration for the phugoid roots, the quadratic is:

$$\lambda^2 + .00564\lambda + .00332 = 0$$

The period of the phugoid is 109 seconds and the time to damp to half amplitude is 246 seconds.

The short period roots were obtained from the remaining factor of the quartic:

$$\lambda^2 + 2.19976\lambda + 6.85419 = 0$$

The period of the short period mode is 2.65 seconds and time to damp to half amplitude is 0.63 seconds.

The complete solution was obtained by using the Reeves Electronic Analog Computer and these results are shown in Fig. 3. The values obtained by the computer method and the stability quartic method are seen to be in excellent agreement and these values will be used as the basis for comparison of the various methods employed for simplification.

## SHORT PERIOD METHOD

It is assumed that the short period mode is essentially a coupling between the perturbations w and q alone. By this method, the equation is:

$$A\lambda^2 + B\lambda + C = 0$$

where:

$$A = 1$$

$$B = 2.201$$

$$C = 6.84$$

From this, the period is 2.66 seconds and the time to damp to half amplitude is 0.63 seconds. This is in excellent agreement with the results obtained from the quartic. The computer solution is shown in Fig. 4.

## PHUGOID METHOD I

The second method to be compared is the previously discussed phugoid simplification in which $\Delta\alpha$ was considered to be equal to $\frac{w}{V_p}$. The quadratic

equation for this method is:

$$A\lambda^2 + B\lambda + C = 0$$

where:

$$A = 8.5306$$

$$B = .04336$$

$$C = .02275$$

The period by this method is 122 seconds and the time to one-half amplitude is 273 seconds. The period is seen to be about 12 percent in error. The computer solution is shown in Fig. 5.

## PHUGOID METHOD II

The third method to be compared is the method which requires that when $\Delta\alpha = 0$, $w = u \tan \alpha$. The quadratic equation which results is:

$$A\lambda^2 + B\lambda + C = 0$$

where:

$$A = 8.56$$

$$B = .0555$$

$$C = .034$$

The method gives a period of 101 seconds and the time to damp to half amplitude is 213 seconds. The computer solution is shown in Fig. 6 for further comparison. It is seen that by this method the period is about eight percent too low.

## MODIFIED PHUGOID METHOD II

The fourth method to be compared is the method evolved in this report. This method requires that when $\Delta\alpha = 0$, $w = u \tan \alpha$. The quadratic equation

is:

$$A\lambda^2 + B\lambda + C = 0$$

where:

$A = 8.92$

$B = .418$

$C = .034$

The period by this method is 110 seconds and the time to damp to half amplitude is 29.6 seconds. The computer solution is shown in Fig. 7 for further comparison. It is seen that by this method the period is accurate to less than one percent.

## PHUGOID METHOD III

The fifth and final numerical solution is the last phugoid case where w is considered to be zero and the second force equation is ignored. This method produces the cubic equation:

$$A\lambda^3 + B\lambda^2 + C\lambda + D = 0$$

where:

$A = 1$

$B = 1.60939$

$C = .01788$

$C = .01106$

This produces one negative real root and a complex pair. The period by this method is 75.7 seconds and the time to damp to half amplitude is 203 seconds. The computer solution is shown in Fig. 8 for further comparison. The error by this method is about 30 percent.

## CASE II

## AIRPLANE DATA AND VELOCITIES

| | |
|---|---|
| $S = 600$ sq ft | $X_u = -6.3$ lb sec/ft |
| $m = 1180$ slugs | $X_w = 165$ lb sec/ft |
| $I_{yy} = 80{,}000$ slug ft$^2$ | $Z_u = -162.6$ lb sec/ft |
| $h =$ sea level | $Z_w = -980$ lb sec/ft |
| $\alpha = 6.5$ degrees $= \Theta_1$ | $M_w = -1345$ lb sec |
| $V_p = 279$ ft/sec | $M_{\dot{w}} = -102.7$ lb sec$^2$ |
| $U_1 = 279$ ft/sec | $M_u = 153.7$ lb sec |
| $W_1 = 31.9$ ft/sec | $Z_q = -4325$ lb sec |
| $M = .25$ | $M_q = -115000$ lb ft sec |

Computer solutions for the second case, Mach 0.25 at sea level are shown in Figs. 9 through 13 and the results are seen to parallel Case I.

## CONCLUSIONS

Comparison of the numerical results and computer solutions of the several methods presented lead to the following conclusions:

1. The concepts concerning the short period symmetric motion of an airplane when applied to the general equations provide an acceptable approximation to the period of that motion.
2. Concepts concerning the period of the phugoid mode, when applied, provide accuracies ranging from 30 percent low to 12 percent high, depending upon the method of application. It was concluded, therefore, that use of the existing simplifications does not accurately describe the period of the true motion because essential couplings

are neglected.

3. The Reeves Electronic Analog Computer provided rapid accurate solutions to many sets of differential equations which had to be solved. The utility of the computer in investigations such as this is unquestionable. Variation of stability derivatives or coupling between equations could be accomplished by changing one or several resistances and the new solution was immediately obtainable.

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3. Duncan, William J., Control and Stability of Aircraft, Cambridge University Press, London, New York.
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6. Lanchester, Frederick W., Aerodonetics, A. Constable and Company, London.

## ANALOG COMPUTER CIRCUIT - SYMMETRIC EQUATIONS

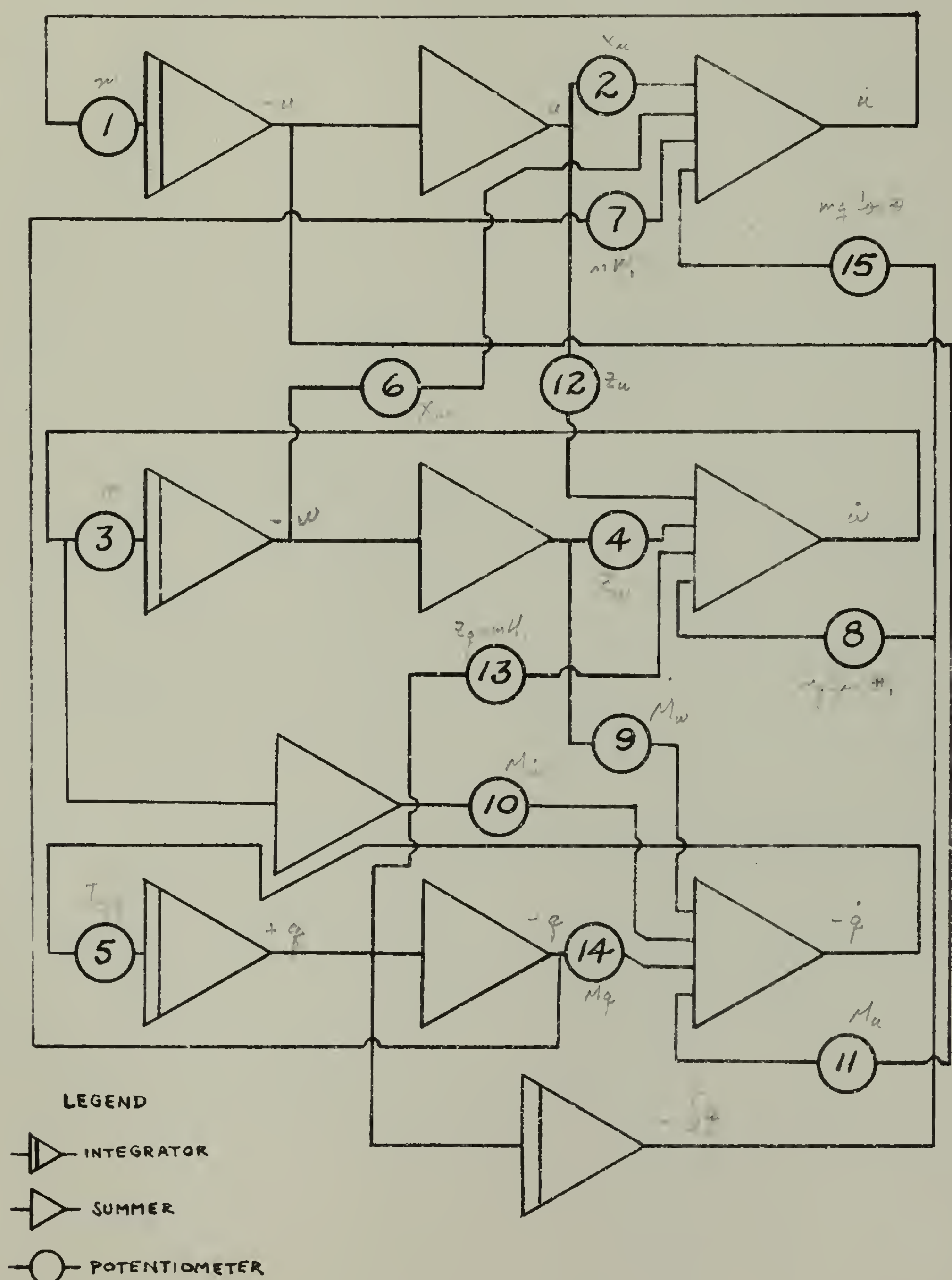
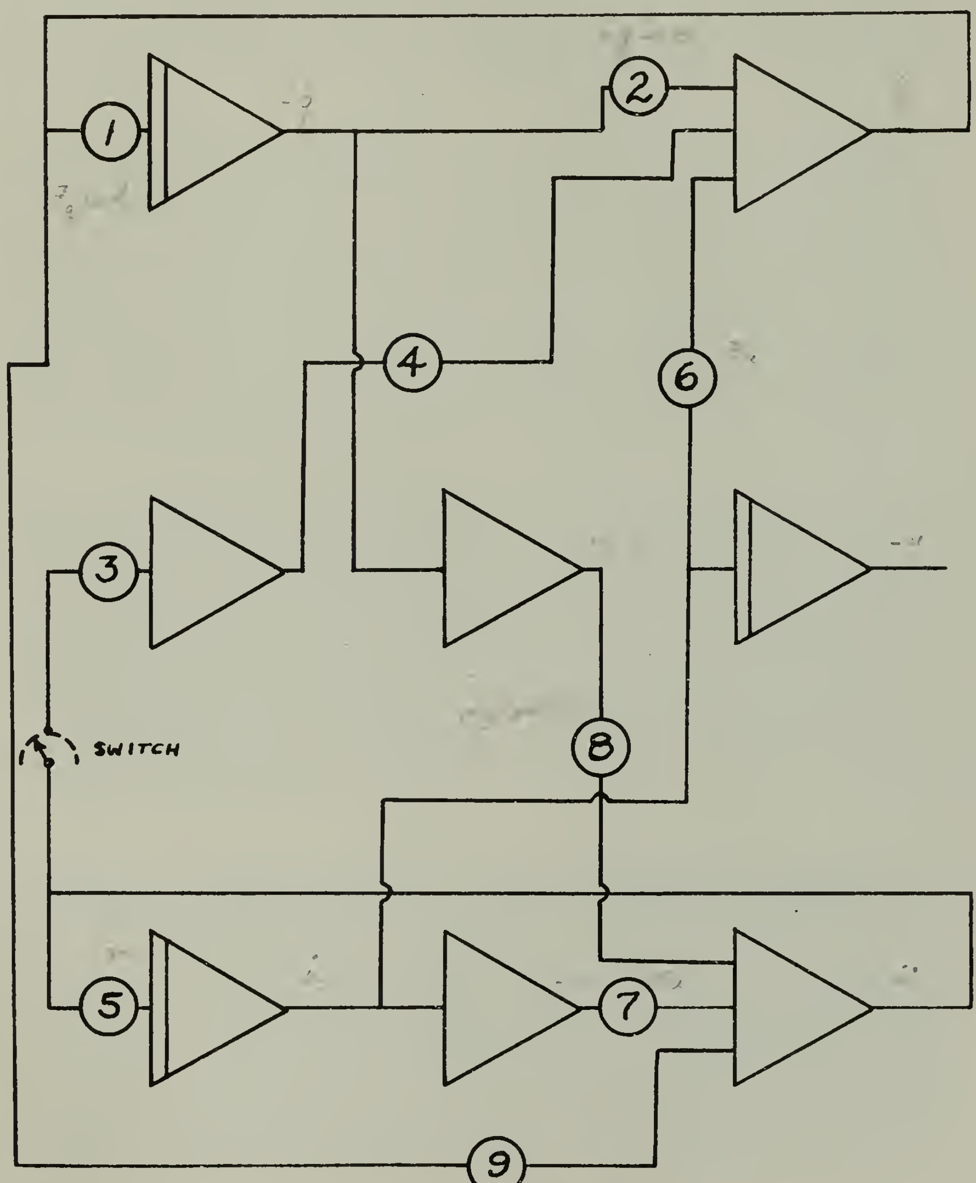



FIG ~ 1

## ANALOG COMPUTER CIRCUIT – SIMPLIFIED EQUATIONS




LEGEND

SEE FIG ~ 1

FIG ~ 2

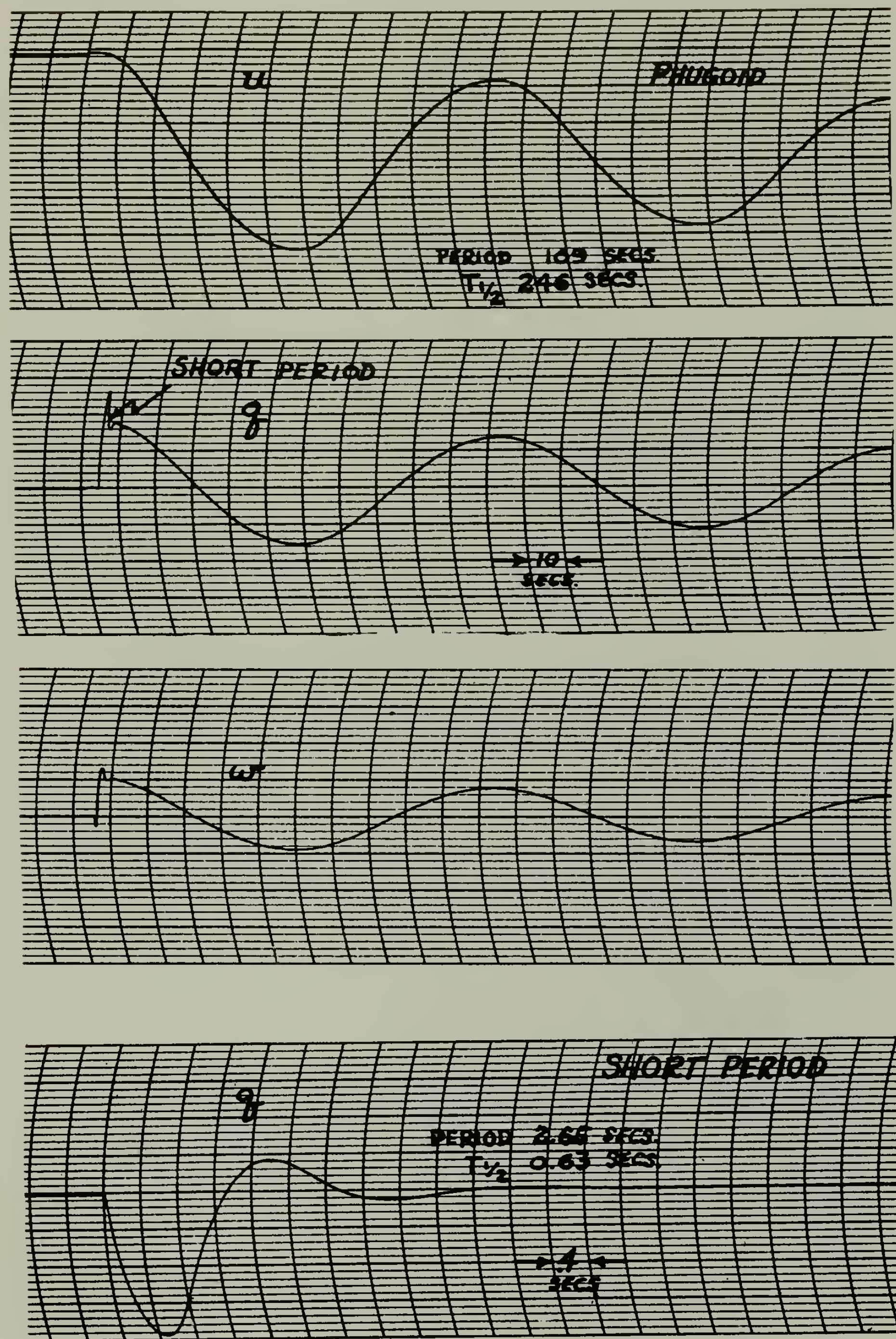



FIG ~ 3

COMPUTER TRACES - COMPLETE EQUATIONS - MACH .75

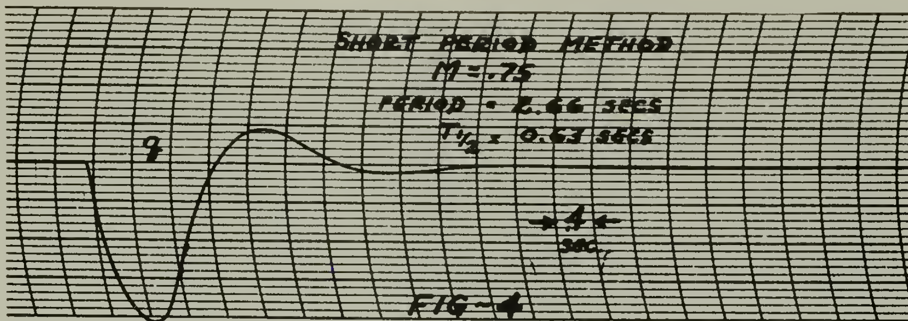



FIG ~ 4

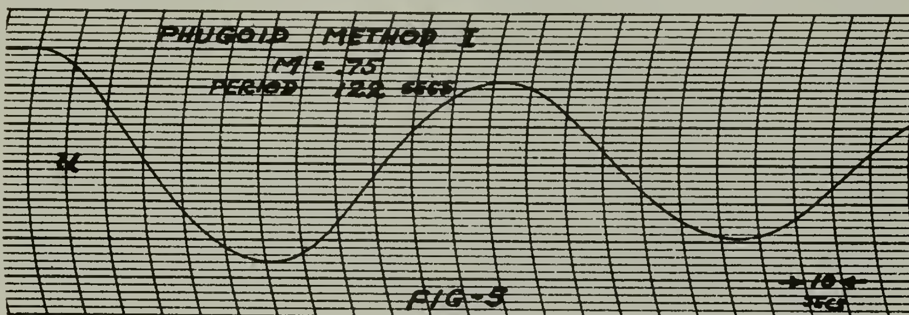



FIG ~ 5

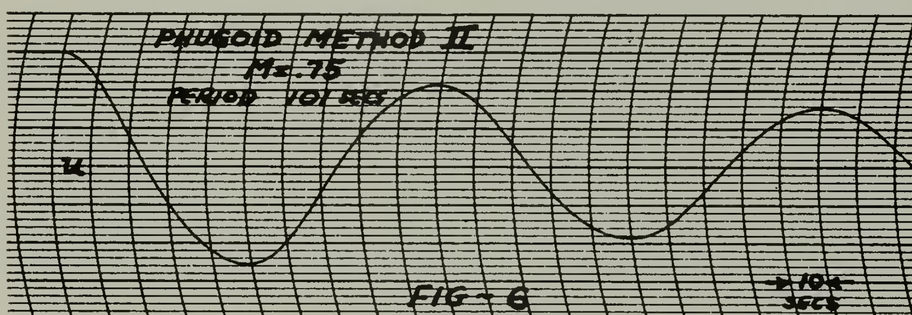



FIG ~ 6

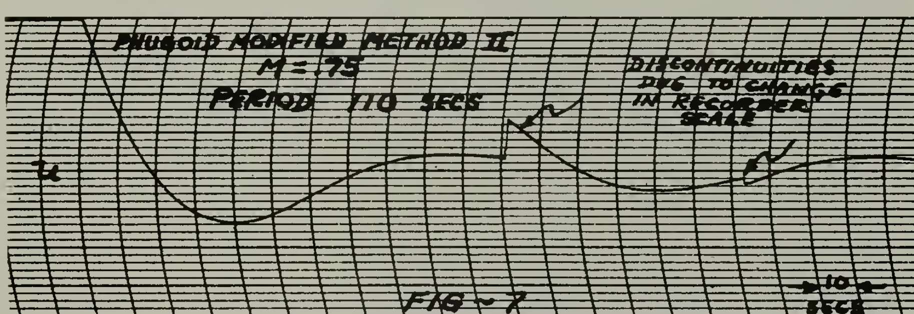



FIG ~ 7

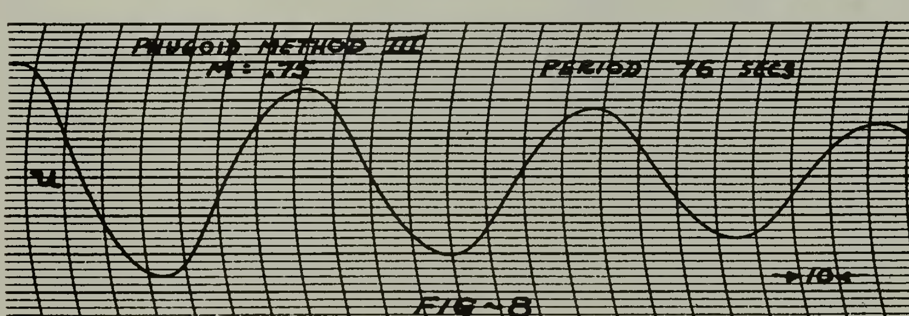



FIG ~ 8

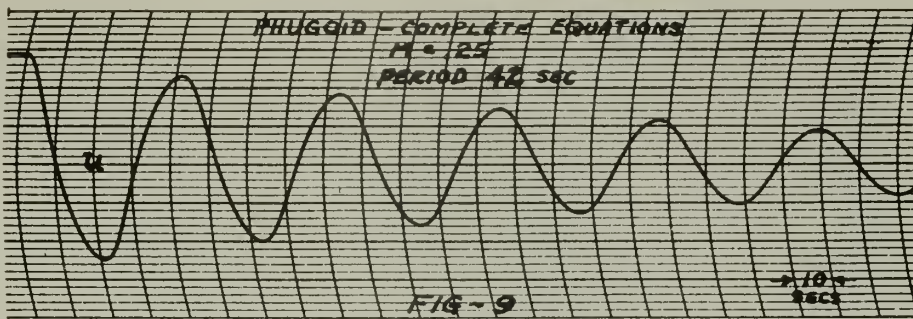


FIG ~ 9

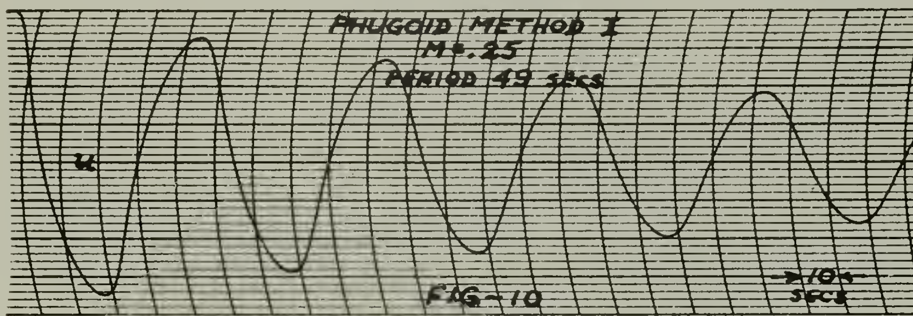


FIG ~ 10

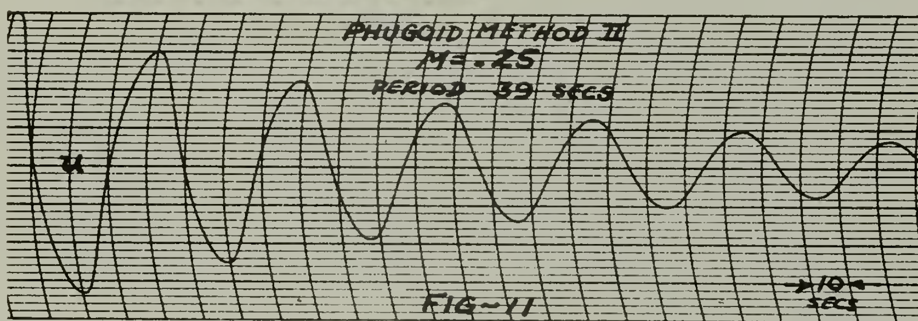


FIG ~ 11

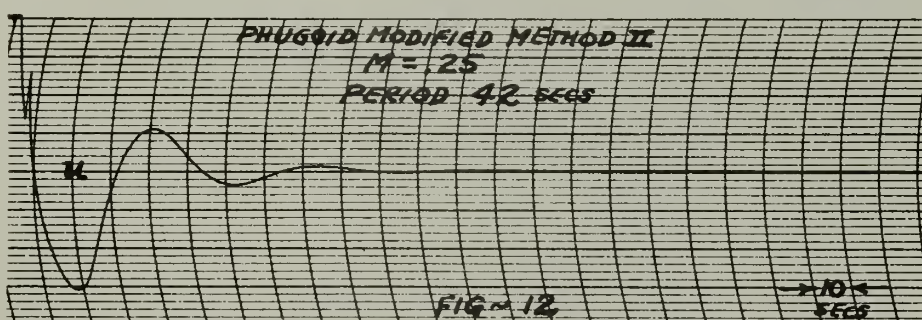


FIG ~ 12

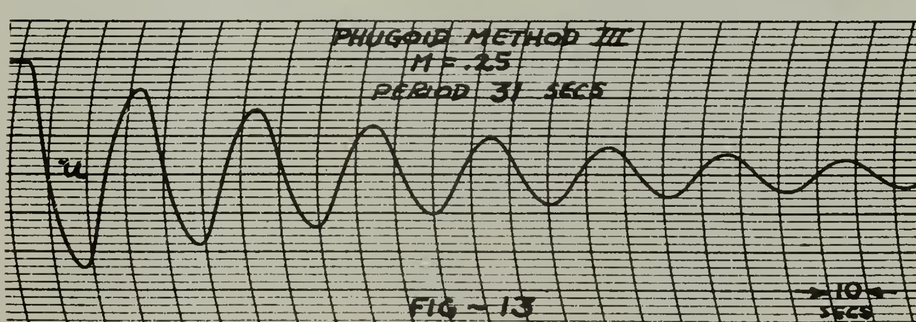


FIG ~ 13

On the validity of certain classical assumptions in aircraft stability.



Kelly
On the validity of certain classical assumptions in aircraft stability.